Integraltabelle

A) Grundintegrale

1) $\displaystyle\int x^n\,dx \;=\; \frac{x^{n+1}}{n+1}$ $\qquad (n \neq -1)$

2) $\displaystyle\int \frac{1}{x}\,dx \;=\; \ln|x|$

3) $\displaystyle\int \frac{1}{1+x^2}\,dx \;=\; \arctan x$

4) $\displaystyle\int \cos x\,dx \;=\; \sin x$

5) $\displaystyle\int \sin x\,dx \;=\; -\cos x$

6) $\displaystyle\int \frac{1}{\cos^2 x}\,dx \;=\; \tan x$

7) $\displaystyle\int \frac{1}{\sin^2 x}\,dx \;=\; -\cot x$

8) $\displaystyle\int e^x\,dx \;=\; e^x$

9) $\displaystyle\int a^x\,dx \;=\; \frac{a^x}{\ln a}$

10) $\displaystyle\int \cosh x\,dx \;=\; \sinh x$

11) $\displaystyle\int \sinh x\,dx \;=\; \cosh x$

12) $\displaystyle\int \frac{1}{\cosh^2 x}\,dx \;=\; \tanh x$

13) $\displaystyle\int \frac{1}{\sinh^2 x}\,dx \;=\; -\coth x$

14) $\displaystyle\int \ln x\,dx \;=\; x\cdot\ln x \,-\, x$

B) Rationale Funktionen

15) $\displaystyle\int (x-a)^n\,dx \;=\; \frac{(x-a)^{n+1}}{n+1}$ $\qquad (n \neq -1)$

16) $\displaystyle\int \frac{1}{x-a}\,dx \;=\; \ln|x-a|$

17) $\displaystyle\int \frac{1}{(x-a)(x-b)}\,dx \;=\; \frac{1}{a-b}\ln\left|\frac{x-a}{x-b}\right|$ $\qquad (a \neq b)$

18) $\displaystyle\int \frac{1}{a^2+x^2}\,dx \;=\; \frac{1}{a}\arctan\frac{x}{a}$

19) $\displaystyle\int \frac{1}{\left(a^2+x^2\right)^2}\,dx \;=\; \frac{x}{2a^2(x^2+a^2)} \,+\, \frac{1}{2a^3}\arctan\frac{x}{a}$

20) $\int \dfrac{1}{a\,x^2 + bx + c}\,dx \quad = \quad \dfrac{2}{\sqrt{\Delta}}\arctan\dfrac{2ax+b}{\sqrt{\Delta}}$ für $\quad \Delta > 0$

$= \quad \dfrac{1}{\sqrt{-\Delta}}\ln\left|\dfrac{2ax+b-\sqrt{-\Delta}}{2ax+b+\sqrt{-\Delta}}\right|$ für $\quad \Delta < 0$

$= \quad \dfrac{-2}{2ax+b}$ für $\quad \Delta = 0$

mit $\qquad \Delta = 4ac - b^2$

21) $\int \dfrac{x}{a\,x^2 + bx + c}\,dx \quad = \quad \dfrac{1}{2a}\ln\left|a\,x^2 + bx + c\right| - \dfrac{b}{2a}\int\dfrac{1}{a\,x^2 + bx + c}\,dx$ (vgl. 20)

Bei Integralen über echt gebrochenrationale Funktionen wird auf die Methode der Partialbruchzerlegung verwiesen.

C) Irrationale Funktionen

22) $\int \sqrt{ax+b}\,dx \quad = \quad \dfrac{2}{3a}(ax+b)^{3/2}$

23) $\int \dfrac{1}{\sqrt{ax+b}}\,dx \quad = \quad \dfrac{2}{a}\sqrt{ax+b}$

24) $\int x\sqrt{ax+b}\,dx \quad = \quad \dfrac{2}{15a^2}(3ax-2b)(ax+b)^{3/2}$

25) $\int \dfrac{x}{\sqrt{ax+b}}\,dx \quad = \quad \dfrac{2}{3a^2}(ax-2b)\sqrt{ax+b}$

26) $\int \dfrac{1}{x\sqrt{ax+b}}\,dx \quad = \quad \dfrac{1}{\sqrt{b}}\ln\left|\dfrac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right|$ für $b > 0$

$= \quad \dfrac{2}{\sqrt{b}}\arctan\sqrt{\dfrac{ax+b}{-b}}$ für $b < 0$

27) $\int \dfrac{1}{x}\sqrt{ax+b}\,dx \quad = \quad 2\sqrt{ax+b} + b\int\dfrac{1}{x\sqrt{ax+b}}\,dx$ (vgl. 26)

28) $\int \sqrt{a^2-x^2}\,dx \quad = \quad \dfrac{1}{2}\left(x\sqrt{a^2-x^2} + a^2\arcsin\dfrac{x}{a}\right)$

29) $\int x\sqrt{a^2-x^2}\,dx \quad = \quad -\dfrac{1}{3}(a^2-x^2)^{3/2}$

30) $\int \dfrac{1}{x}\sqrt{a^2-x^2}\,dx \quad = \quad \sqrt{a^2-x^2} - a\ln\left|\dfrac{a+\sqrt{a^2-x^2}}{x}\right|$

31) $\int \dfrac{1}{\sqrt{a^2-x^2}}\,dx \quad = \quad \arcsin\dfrac{x}{a}$

32) $\int \dfrac{x}{\sqrt{a^2-x^2}}\,dx \quad = \quad -\sqrt{a^2-x^2}$

33) $\displaystyle\int \frac{x^2}{\sqrt{a^2-x^2}}\,dx \;=\; \frac{1}{2}\left(-x\sqrt{a^2-x^2}+a^2\arcsin\frac{x}{a}\right)$

34) $\displaystyle\int \sqrt{a^2+x^2}\,dx \;=\; \frac{1}{2}\left(x\sqrt{a^2+x^2}+a^2\,\mathrm{ar\,sinh}\frac{x}{a}\right)$

$\displaystyle\qquad\qquad\qquad\quad =\; \frac{1}{2}\left(x\sqrt{a^2+x^2}+a^2\ln(x+\sqrt{a^2+x^2}\,)\right)$

35) $\displaystyle\int x\sqrt{a^2+x^2}\,dx \;=\; \frac{1}{3}(a^2+x^2)^{3/2}$

36) $\displaystyle\int \frac{1}{x}\sqrt{a^2+x^2}\,dx \;=\; \sqrt{a^2+x^2}\;-\;a\ln\left|\frac{a+\sqrt{a^2+x^2}}{x}\right|$

37) $\displaystyle\int \frac{1}{\sqrt{a^2+x^2}}\,dx \;=\; \mathrm{ar\,sinh}\frac{x}{a} \;=\; \ln(x+\sqrt{a^2+x^2}\,)$

38) $\displaystyle\int \frac{x}{\sqrt{a^2+x^2}}\,dx \;=\; \sqrt{a^2+x^2}$

39) $\displaystyle\int \frac{x^2}{\sqrt{a^2+x^2}}\,dx \;=\; \frac{1}{2}\left(x\sqrt{a^2+x^2}-a^2\,\mathrm{ar\,sinh}\frac{x}{a}\right)$

$\displaystyle\qquad\qquad\qquad\quad =\; \frac{1}{2}\left(x\sqrt{a^2+x^2}-a^2\ln(x+\sqrt{a^2+x^2}\,)\right)$

40) $\displaystyle\int \frac{1}{x\sqrt{a^2+x^2}}\,dx \;=\; -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2+x^2}}{x}\right|$

41) $\displaystyle\int \frac{1}{x^2\sqrt{a^2+x^2}}\,dx \;=\; -\frac{\sqrt{a^2+x^2}}{a^2 x}$

42) $\displaystyle\int \sqrt{x^2-a^2}\,dx \;=\; \frac{1}{2}\left(x\sqrt{x^2-a^2}-a^2\,\mathrm{ar\,cosh}\frac{x}{a}\right)$

$\displaystyle\qquad\qquad\qquad\quad =\; \frac{1}{2}\left(x\sqrt{x^2-a^2}-a^2\ln\left|x+\sqrt{x^2-a^2}\right|\right)$

43) $\displaystyle\int x\sqrt{x^2-a^2}\,dx \;=\; \frac{1}{3}(x^2-a^2)^{3/2}$

44) $\displaystyle\int \frac{1}{x}\sqrt{x^2-a^2}\,dx \;=\; \sqrt{x^2-a^2}\;-\;a\arccos\frac{a}{x}$

45) $\displaystyle\int \frac{1}{\sqrt{x^2-a^2}}\,dx \;=\; \mathrm{ar\,cosh}\frac{x}{a} \;=\; \ln\left|x+\sqrt{x^2-a^2}\right|$

46) $\displaystyle\int \frac{x}{\sqrt{x^2-a^2}}\,dx \;=\; \sqrt{x^2-a^2}$

47) $\displaystyle\int \frac{x^2}{\sqrt{x^2-a^2}}\,dx \;=\; \frac{1}{2}\left(x\sqrt{x^2-a^2}-a^2\,\mathrm{ar\,cosh}\frac{x}{a}\right)$

$\displaystyle\qquad\qquad\qquad\quad =\; \frac{1}{2}\left(x\sqrt{x^2-a^2}-a^2\ln\left|x+\sqrt{x^2-a^2}\right|\right)$

48) $\int \dfrac{1}{\sqrt{a\,x^2 + bx + c}}\, dx = \dfrac{1}{\sqrt{a}} \ln\left| 2\sqrt{a(a\,x^2 + bx + c)} + 2ax + b\right|$ für $a > 0$

$\phantom{48) \int \dfrac{1}{\sqrt{a\,x^2 + bx + c}}\, dx} = \dfrac{-1}{\sqrt{-a}} \arcsin \dfrac{2ax + b}{\sqrt{b^2 - 4ac}}$ für $a < 0$

49) $\int \dfrac{x}{\sqrt{a\,x^2 + bx + c}}\, dx = \dfrac{1}{a}\sqrt{a\,x^2 + bx + c} - \dfrac{b}{2a}\int \dfrac{1}{\sqrt{a\,x^2 + bx + c}}\, dx$ (vgl. 48)

50) $\int \sqrt{a\,x^2 + bx + c}\, dx = \dfrac{2ax + b}{4a}\sqrt{a\,x^2 + bx + c} + \dfrac{4ac - b^2}{8a}\int \dfrac{1}{\sqrt{a\,x^2 + bx + c}}\, dx$

D) Trigonometrische Funktionen

51) $\int \sin ax \, dx = -\dfrac{1}{a}\cos ax$

52) $\int \sin^2 ax \, dx = \dfrac{x}{2} - \dfrac{1}{4a}\sin 2ax$

53) $\int \sin^3 ax \, dx = -\dfrac{1}{a}\cos ax + \dfrac{1}{3a}\cos^3 ax$

54) $\int \sin^n ax \, dx = -\dfrac{\sin^{n-1} ax \cdot \cos ax}{n \cdot a} + \dfrac{n-1}{n}\int \sin^{n-2} ax \, dx$

55) $\int x \cdot \sin ax \, dx = \dfrac{\sin ax}{a^2} - \dfrac{x \cdot \cos ax}{a}$

56) $\int x^n \cdot \sin ax \, dx = -\dfrac{x^n}{a}\cos ax + \dfrac{n}{a}\int x^{n-1} \cdot \cos ax \, dx$

57) $\int \dfrac{1}{\sin ax}\, dx = \dfrac{1}{a}\ln\left|\tan \dfrac{ax}{2}\right|$

58) $\int \dfrac{1}{\sin^n ax}\, dx = -\dfrac{1}{a(n-1)} \cdot \dfrac{\cos ax}{\sin^{n-1} ax} + \dfrac{n-2}{n-1}\int \dfrac{1}{\sin^{n-2} ax}\, dx$ $(n > 1)$

59) $\int \dfrac{1}{1 + \sin ax}\, dx = \dfrac{1}{a}\tan\left(\dfrac{ax}{2} - \dfrac{\pi}{4}\right)$

60) $\int \dfrac{1}{1 - \sin ax}\, dx = \dfrac{1}{a}\tan\left(\dfrac{ax}{2} + \dfrac{\pi}{4}\right)$

61) $\int \sin ax \cdot \sin bx \, dx = \dfrac{\sin(a-b)x}{2(a-b)} - \dfrac{\sin(a+b)x}{2(a+b)}$ $(\,|a| \neq |b|$

62) $\int \cos ax\, dx = \dfrac{1}{a} \sin ax$

63) $\int \cos^2 ax\, dx = \dfrac{x}{2} + \dfrac{1}{4a} \sin 2ax$

64) $\int \cos^3 ax\, dx = \dfrac{1}{a} \sin ax - \dfrac{1}{3a} \sin^3 ax$

65) $\int \cos^n ax\, dx = -\dfrac{\cos^{n-1} ax \cdot \sin ax}{n \cdot a} + \dfrac{n-1}{n} \int \cos^{n-2} ax\, dx$

66) $\int x \cdot \cos ax\, dx = \dfrac{\cos ax}{a^2} + \dfrac{x \cdot \sin ax}{a}$

67) $\int x^n \cdot \cos ax\, dx = \dfrac{x^n}{a} \sin ax - \dfrac{n}{a} \int x^{n-1} \cdot \sin ax\, dx$

68) $\int \dfrac{1}{\cos ax}\, dx = \dfrac{1}{a} \ln \left| \tan \left(\dfrac{ax}{2} + \dfrac{\pi}{4} \right) \right|$

69) $\int \dfrac{1}{\cos^n ax}\, dx = \dfrac{1}{a(n-1)} \cdot \dfrac{\sin ax}{\cos^{n-1} ax} + \dfrac{n-2}{n-1} \int \dfrac{1}{\cos^{n-2} ax}\, dx$ $\quad (n > 1)$

70) $\int \dfrac{1}{1 + \cos ax}\, dx = \dfrac{1}{a} \tan \dfrac{ax}{2}$

71) $\int \dfrac{1}{1 - \cos ax}\, dx = -\dfrac{1}{a} \cot \dfrac{ax}{2}$

72) $\int \cos ax \cdot \cos bx\, dx = \dfrac{\sin(a-b)x}{2(a-b)} + \dfrac{\sin(a+b)x}{2(a+b)}$ $\quad (|a| \neq |b|)$

73) $\int \sin ax \cdot \cos ax\, dx = \dfrac{1}{2a} \sin^2 ax$

74) $\int \sin ax \cdot \cos bx\, dx = -\dfrac{\cos(a-b)x}{2(a-b)} - \dfrac{\cos(a+b)x}{2(a+b)}$ $\quad (|a| \neq |b|)$

75) $\int \tan ax\, dx = -\dfrac{1}{a} \ln |\cos ax|$

76) $\int \tan^2 ax\, dx = \dfrac{1}{a} \tan ax - x$

77) $\int \tan^n ax\, dx = \dfrac{1}{a \cdot (n-1)} \tan^{n-1} ax - \int \tan^{n-2} ax\, dx$ $\quad (n \neq 1)$

78) $\int \cot ax\, dx = \dfrac{1}{a} \ln |\sin ax|$

79) $\int \cot^n ax\, dx = -\dfrac{1}{a \cdot (n-1)} \cot^{n-1} ax - \int \cot^{n-2} ax\, dx$ $\quad (n \neq 1)$

E) Exponential- und Hyperbelfunktionen

80) $\int e^{ax}\, dx = \dfrac{1}{a} e^{ax}$

81) $\int x e^{ax}\, dx = \dfrac{1}{a^2} e^{ax} (ax - 1)$

82) $\int x^n e^{ax} \, dx = \dfrac{1}{a} x^n e^{ax} - \dfrac{n}{a} \int x^{n-1} e^{ax} \, dx$

83) $\int \sinh ax \, dx = \dfrac{1}{a} \cosh ax$

84) $\int \sinh^2 ax \, dx = \dfrac{1}{4a} \sinh 2ax - \dfrac{x}{2}$

85) $\int \dfrac{1}{\sinh ax} \, dx = \dfrac{1}{a} \ln \left| \tanh \dfrac{ax}{2} \right|$

86) $\int \cosh ax \, dx = \dfrac{1}{a} \sinh ax$

87) $\int \cosh^2 ax \, dx = \dfrac{1}{4a} \sinh 2ax + \dfrac{x}{2}$

88) $\int \dfrac{1}{\cosh ax} \, dx = \dfrac{2}{a} \arctan e^{ax}$

89) $\int \tanh ax \, dx = \dfrac{1}{a} \ln \cosh ax$

90) $\int \coth ax \, dx = \dfrac{1}{a} \ln \left| \sinh ax \right|$

F) Gemischte Ausdrücke aus D) und E)

91) $\int e^{ax} \sin bx \, dx = \dfrac{e^{ax}}{a^2 + b^2} \left(a \sin bx - b \cos bx \right)$

92) $\int e^{ax} \cos bx \, dx = \dfrac{e^{ax}}{a^2 + b^2} \left(a \cos bx + b \sin bx \right)$

G) Logarithmusfunktionen

93) $\int \ln x \, dx = x \cdot \ln x - x$

94) $\int \ln^2 x \, dx = x \cdot \ln^2 x - 2x \ln x + 2x$

95) $\int x^m \ln x \, dx = x^{m+1} \left(\dfrac{\ln x}{m+1} - \dfrac{1}{(m+1)^2} \right)$ $\qquad (m \neq -1)$

96) $\int \dfrac{1}{x} \ln x \, dx = \dfrac{1}{2} \ln^2 x$

H) Arcusfunktionen

97) $\int \arcsin \dfrac{x}{a} \, dx = x \arcsin \dfrac{x}{a} + \sqrt{a^2 - x^2}$

98) $\int \arccos \dfrac{x}{a} \, dx = x \arccos \dfrac{x}{a} - \sqrt{a^2 - x^2}$

99) $\int \arctan \dfrac{x}{a} \, dx = x \arctan \dfrac{x}{a} - \dfrac{a}{2} \ln (a^2 + x^2)$

100) $\int \text{arc cot} \dfrac{x}{a} \, dx = x \, \text{arc cot} \dfrac{x}{a} + \dfrac{a}{2} \ln (a^2 + x^2)$