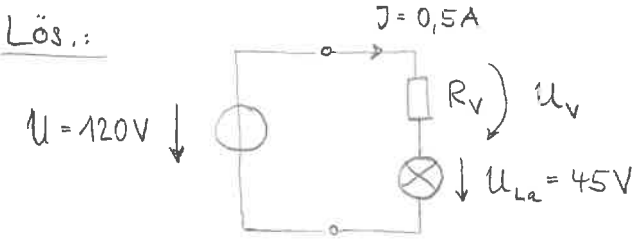
3. Schaltungsanalyse

3.1) Geg.: $U = 120\,V$; $U_{La} = 45\,V$; $J = 0{,}5\,A$;

Ges.: $R_V = ?$

Lös.:



$U = U_V + U_{La}$

$U_V = U - U_{La} = 120\,V - 45\,V = 75\,V$;

$R_V = \frac{U_V}{J} = \frac{75\,V}{0{,}5\,A} = 150\,\Omega$;

3.2) Geg.: Leitungswiderstand R_L: $d = 1{,}5\,mm$; $l = 200\,m$ (bzw. $l = 400\,m$ für Hin- u. Rückleitung)

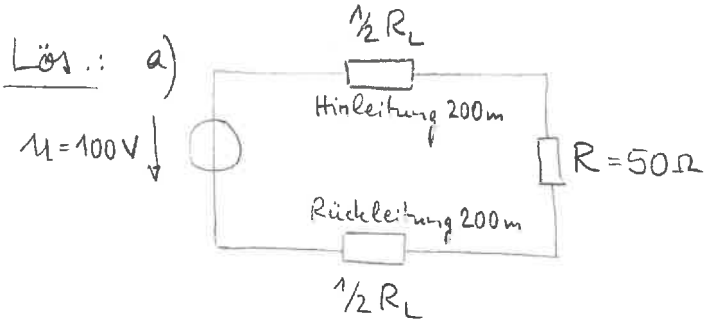
$\varrho_{Cu} = 0{,}018\,\frac{\Omega \cdot mm^2}{m}$;

Verbraucherwiderstand $R = 50\,\Omega$;

Quellenspannung $U = 100\,V$;

Ges.: a) Schaltung b) $R_L = ?$; $U_R = ?$; u_L in % = ? ;

Lös.: a)



$A = \frac{d^2 \cdot \pi}{4} = \frac{(1{,}5\,mm)^2 \cdot \pi}{4} = 1{,}767\,mm^2$;

b) $R_L = \frac{\varrho_{Cu} \cdot l}{A} = \frac{0{,}018\,\frac{\Omega \cdot mm^2}{m} \cdot 400\,m}{1{,}767\,mm^2} = 4{,}075\,\Omega$;

$J = \frac{U}{R_L + R} = \frac{100\,V}{4{,}075\,\frac{V}{A} + 50\,\frac{V}{A}} = 1{,}850\,A$;

$U_R = J \cdot R = 1{,}850\,A \cdot 50\,\frac{V}{A} = 92{,}5\,V$;

$U_L = U - U_R = 100\,V - 92{,}5\,V = 7{,}5\,V$;

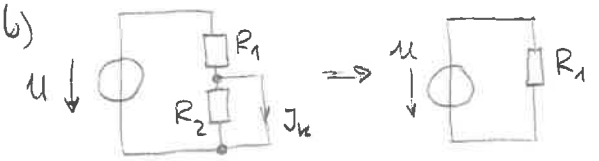
$u_L = \frac{U_L}{U} \cdot 100\,\% = \frac{7{,}5\,V}{100\,V} \cdot 100\,\% = 7{,}5\,\%$;

3. Schaltungsanalyse

3.3) Geg.: $U = 60\,V$; $U_2 = 10\,V$; $J_K = 1{,}0\,A$;

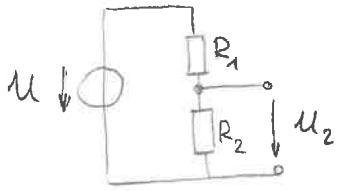
Ges.: R_1 und R_2

Lös.: aus Bild b)



$$R_1 = \frac{U}{J_K} = \frac{60\,V}{1{,}0\,A} = 60\,\Omega \;;$$

Spannungsteilerregel:



$$\frac{U_2}{U} = \frac{R_2}{R_1 + R_2}$$

$$\frac{R_1 + R_2}{R_2} = \frac{U}{U_2} \rightsquigarrow$$

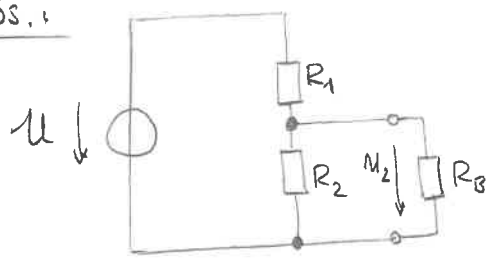
$$\frac{R_1}{R_2} + 1 = \frac{U}{U_2} \rightsquigarrow \frac{R_1}{R_2} = \frac{U}{U_2} - 1 \rightsquigarrow R_2 = \frac{R_1}{\frac{U}{U_2} - 1}$$

$$\rightsquigarrow R_2 = \frac{R_1}{\frac{U}{U_2} - 1} = \frac{60\,\Omega}{\frac{60\,V}{10\,V} - 1} = 12\,\Omega \;;$$

3.4) Geg.: $U = 100\,V$; $R = R_1 + R_2 = 400\,\Omega$; $R_B = 800\,\Omega$; $U_2 = 40\,V$;

Ges.: R_1 und R_2

Lös.:



$$R_{2\|B} = \frac{R_2 \cdot R_B}{R_2 + R_B}$$

Spannungsteilerregel:

$$\frac{U_2}{U} = \frac{R_{2\|B}}{R_1 + R_{2\|B}}$$

$$\rightsquigarrow U_2 = \frac{\frac{R_2 \cdot R_B}{R_2 + R_B}}{R_1 + \frac{R_2 \cdot R_B}{R_2 + R_B}} \cdot U = \frac{R_2 \cdot R_B}{R_1 \cdot R_2 + R_1 \cdot R_B + R_2 \cdot R_B} \;;$$

mit $R_1 = R - R_2$:

$$U_2 = \frac{R_2 \cdot R_B}{(R - R_2) \cdot R_2 + (R - R_2) \cdot R_B + R_2 \cdot R_B} \cdot U \;;$$

3. Schaltungsanalyse

zu 3.4) $$\frac{R \cdot R_2 - R_2^2 + R \cdot R_B - R_2 R_B + R_2 \cdot R_B}{R_2 \cdot R_B} = \frac{U}{U_2}$$

$$-R_2^2 + R \cdot R_2 + R \cdot R_B = \frac{U}{U_2} \cdot R_B \cdot R_2$$

$$R_2^2 + \frac{U}{U_2} \cdot R_B \cdot R_2 - R \cdot R_2 - R \cdot R_B = 0$$

$$R_2^2 + \left(\frac{U}{U_2} \cdot R_B - R\right) \cdot R_2 - R \cdot R_B = 0$$

$$R_2^2 + \left(\frac{100V}{40V} \cdot 800\Omega - 400\Omega\right) \cdot R_2 - 400\Omega \cdot 800\Omega = 0$$

$$R_2^2 + 1600\Omega \cdot R_2 - 320000\Omega^2 = 0$$

$$ax^2 + bx + c = 0 \qquad x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$R_2 = \frac{-1600\Omega \pm \sqrt{(1600\Omega)^2 + 4 \cdot 1 \cdot 320000\Omega^2}}{2 \cdot 1} = \frac{-1600\Omega \pm \sqrt{3{,}84 \cdot 10^6 \Omega^2}}{2} = \frac{-1600\Omega \pm 1959{,}6\Omega}{2} = \underline{180\Omega};$$

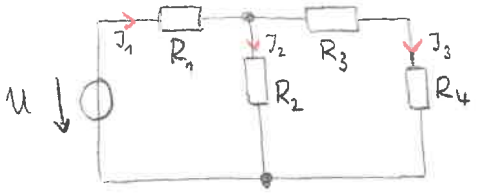
$$R_1 = R - R_2 = 400\Omega - 180\Omega = \underline{220\Omega};$$

3.5) Geg.: $R_1 = 20\Omega$; $R_2 = 30\Omega$; $R_3 = 10\Omega$; $R_4 = 50\Omega$; $U = 12V$;

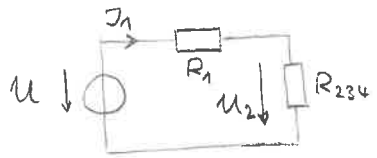
Ges.: J_1, J_2 und J_3

Lös.:

$R_2 \parallel R_3 + R_4$



$$R_{234} = \frac{R_2 \cdot (R_3 + R_4)}{R_2 + R_3 + R_4} = \frac{30\Omega \cdot (10\Omega + 50\Omega)}{30\Omega + 10\Omega + 50\Omega} = \underline{20\Omega};$$



$$J_1 = \frac{U}{R_1 + R_{234}} = \frac{12V}{20\Omega + 20\Omega} = 0{,}3A = \underline{300mA};$$

$$U_2 = J_1 \cdot R_{234} = 0{,}3A \cdot 20\frac{V}{A} = \underline{6{,}0V};$$

$$J_2 = \frac{U_2}{R_2} = \frac{6{,}0V}{30\frac{V}{A}} = 0{,}2A = \underline{200mA}; \qquad J_1 = J_2 + J_3$$

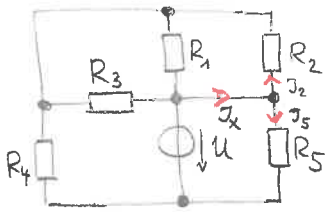
$$\rightsquigarrow J_3 = J_1 - J_2 = 0{,}3A - 0{,}2A = 0{,}1A = \underline{100mA};$$

3. Schaltungsanalyse

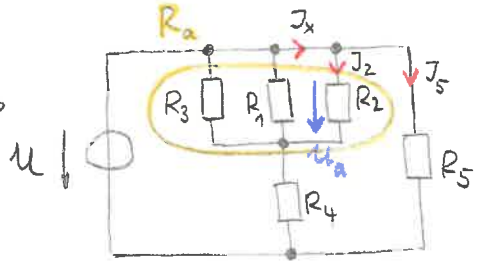
3.6) Geg.: $R_1 = 50\,\Omega$; $R_2 = 45\,\Omega$; $R_3 = 40\,\Omega$; $R_4 = 55\,\Omega$; $R_5 = 60\,\Omega$; $U = 48\,V$;

Ges.: $J_x = ?$

Lös.:



R_1 u. R_2 u. R_3 liegen parallel! ⟹



$$R_a = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{50}+\frac{1}{45}+\frac{1}{40}}\,\Omega = 14{,}9\,\Omega;$$

Spannungsteiler:

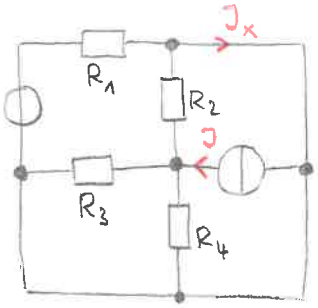
$$\frac{U_a}{U} = \frac{R_a}{R_4+R_a} \rightsquigarrow U_a = U\cdot\frac{R_a}{R_4+R_a} = 48\,V\cdot\frac{14{,}9\,\Omega}{55\,\Omega+14{,}9\,\Omega} = 10{,}2\,V;$$

$$J_x = J_2 + J_5 = \frac{U_a}{R_2} + \frac{U}{R_5} = \frac{10{,}2\,V}{45\,\frac{V}{A}} + \frac{48\,V}{60\,\frac{V}{A}} = 1{,}03\,A;$$

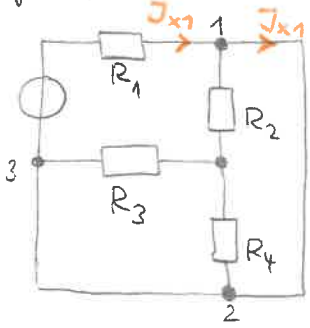
3.7) Geg.: $U = 36\,V$; $J = 2{,}4\,A$; $R_1 = 30\,\Omega$; $R_2 = 50\,\Omega$; $R_3 = 40\,\Omega$; $R_4 = 60\,\Omega$;

Ges.: $J_x = ?$

Lös.:



- der von Stromquelle gelieferte Strom $J = 0$;

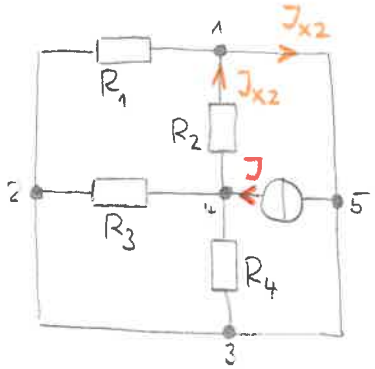


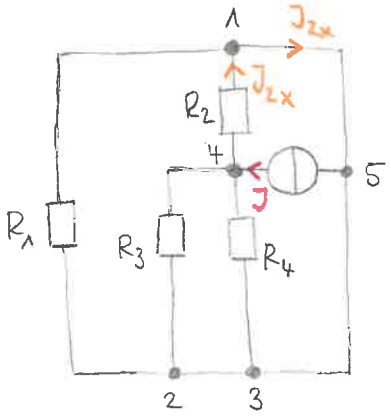
- die Knotenpunkte 1, 2 u. 3 liegen auf dem gleichen elektrischen Potential, damit sind die Widerstände R_2, R_3 u. R_4 überflüssig;
- der Widerstand R_1 liegt an der Spannung U;

$$J_{x1} = \frac{U}{R_1} = \frac{36\,V}{30\,\frac{V}{A}} = 1{,}20\,A;$$

3. Schaltungsanalyse

zu 3.7) - danach wird die von der Spannungsquelle gelieferte Spannung $U = 0$ gesetzt;





- die Widerstände R_2, R_3 u. R_4 sind parallel;
- der Widerstand R_1 ist kurzgeschlossen (die Knotenpunkte 1+5+3+2 liegen auf dem gleichen Potential)

$\Rightarrow$ R_1 ist ohnelos;

$$R_{34} = \frac{R_3 \cdot R_4}{R_3 + R_4} = \frac{40\,\Omega \cdot 60\,\Omega}{40\,\Omega + 60\,\Omega} = 24\,\Omega;$$

Stromteilerregel: R_{ges}

$$J \cdot \frac{R_{34} \cdot R_2}{R_{34} + R_2} = J_{x2} \cdot R_2$$

$$\rightsquigarrow \quad J \cdot \frac{R_{34}}{R_{34} + R_2} = J_{x2} \quad \rightsquigarrow \quad \underline{J_{x2}} = 2{,}4\,A \cdot \frac{24\,\Omega}{50\,\Omega + 24\,\Omega} = \underline{0{,}78\,A};$$

- beide Ströme J_{x1} und J_{x2} werden überlagert:

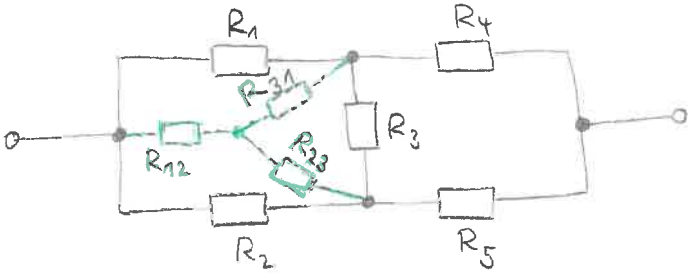
$$\underline{J_x} = J_{x1} + J_{x2} = 1{,}20\,A + 0{,}78\,A = \underline{1{,}98\,A};$$

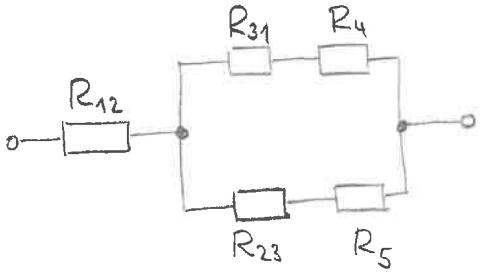
3. Schaltungsanalyse

3.8) Geg.: $R_1 = 55\,\Omega$; $R_2 = 40\,\Omega$; $R_3 = 45\,\Omega$; $R_4 = 40\,\Omega$; $R_5 = 60\,\Omega$;

Ges.: Ersatzwiderstand R ?

Lös.:





Stern – Dreieck – Umwandlung:

$$R_{12} = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{55\,\Omega \cdot 40\,\Omega}{55\,\Omega + 40\,\Omega + 45\,\Omega} = 15{,}7\,\Omega \;;$$

$$R_{23} = \frac{R_2 R_3}{R_1 + R_2 + R_3} = \frac{40\,\Omega \cdot 45\,\Omega}{55\,\Omega + 40\,\Omega + 45\,\Omega} = 12{,}9\,\Omega \;;$$

$$R_{31} = \frac{R_3 R_1}{R_1 + R_2 + R_3} = \frac{45\,\Omega \cdot 55\,\Omega}{55\,\Omega + 40\,\Omega + 45\,\Omega} = 17{,}7\,\Omega \;;$$

$$R = R_{12} \cdot \frac{(R_{31} + R_4) \cdot (R_{23} + R_5)}{(R_{31} + R_4) + (R_{23} + R_5)} =$$

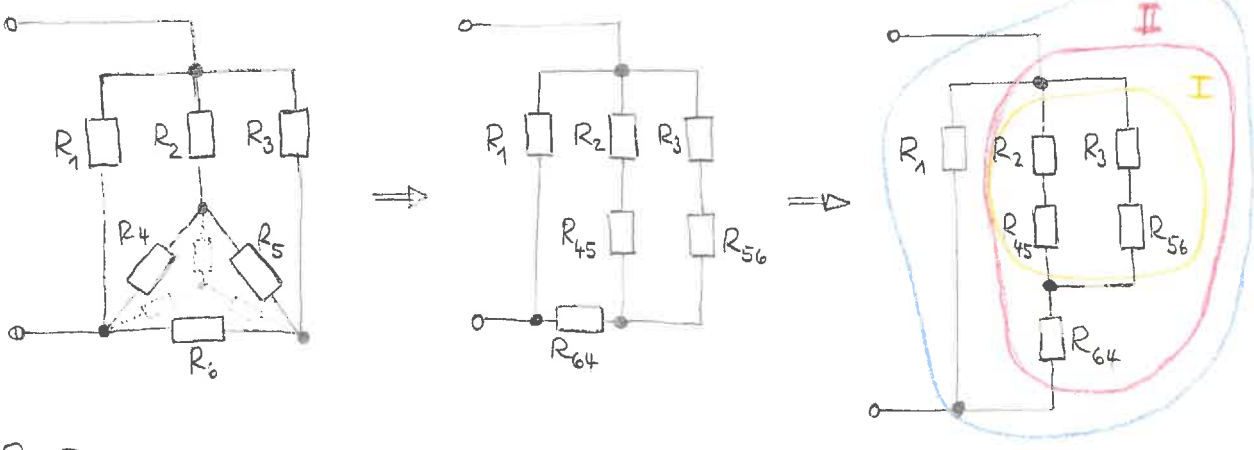
$$= 15{,}7\,\Omega \cdot \frac{(17{,}7\,\Omega + 40\,\Omega) \cdot (12{,}9\,\Omega + 60\,\Omega)}{17{,}7\,\Omega + 40\,\Omega + 12{,}9\,\Omega + 60\,\Omega} = 50{,}8\,\Omega \;;$$

3. Schaltungsanalyse

3.9) Geg.: $R_1 = 1{,}0\,k\Omega$; $R_2 = 1{,}5\,k\Omega$; $R_3 = 2{,}0\,k\Omega$;
$R_4 = 3{,}0\,k\Omega$; $R_5 = 2{,}0\,k\Omega$; $R_6 = 2{,}5\,k\Omega$;

Ges.: Ersatzwiderstand R ?

Lös.:



$$R_{45} = \frac{R_4 R_5}{R_4 + R_5 + R_6} = \frac{3{,}0\,k\Omega \cdot 2.0\,k\Omega}{3{,}0\,k\Omega + 2{,}0\,k\Omega + 2{,}5\,k\Omega} = 0{,}8\,k\Omega ;$$

$$R_{56} = \frac{R_5 R_6}{R_4 + R_5 + R_6} = \frac{2{,}0\,k\Omega \cdot 2{,}5\,k\Omega}{3{,}0\,k\Omega + 2{,}0\,k\Omega + 2{,}5\,k\Omega} = 0{,}67\,k\Omega ;$$

$$R_{64} = \frac{R_6 R_4}{R_4 + R_5 + R_6} = \frac{2{,}5\,k\Omega \cdot 3{,}0\,k\Omega}{3{,}0\,k\Omega + 2{,}0\,k\Omega + 2{,}5\,k\Omega} = 1{,}0\,k\Omega ;$$

I.) $$R_I = \frac{(R_2 + R_{45}) \cdot (R_3 + R_{56})}{R_2 + R_{45} + R_3 + R_{56}} = \frac{(1{,}5\,k\Omega + 0{,}8\,k\Omega) \cdot (2{,}0\,k\Omega + 0{,}67\,k\Omega)}{1{,}5\,k\Omega + 0{,}8\,k\Omega + 2{,}0\,k\Omega + 0{,}67\,k\Omega} = 1{,}235\,k\Omega ;$$

II.) $$R_{II} = R_I + R_{64} = 1{,}235\,k\Omega + 1{,}0\,k\Omega = 2{,}235\,k\Omega ;$$

III.) $$R_{III} = \frac{R_1 \cdot R_{II}}{R_1 + R_{II}} = \frac{1{,}0\,k\Omega \cdot 2{,}235\,k\Omega}{1{,}0\,k\Omega + 2{,}235\,k\Omega} = 0{,}691\,k\Omega = 691\,\Omega ;$$

$$\Rightarrow \quad R = R_{III} = 691\,\Omega ;$$