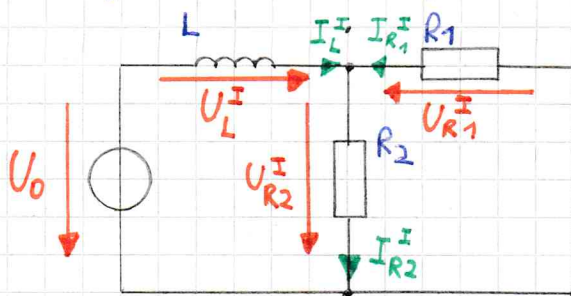


geg: $U_0 = 10V$ $U_1 = 20V \angle 30^\circ$ $f = 10\text{ kHz}$
 $R_1 = 12\Omega$ $R_2 = 30\Omega$ $L = 159,155\mu\text{H}$

ges: a) I_{R2} b) S_0, P_0, Q_0

Lösungsweg 1: Überlagerungssatz

a) I) U_0 aktiv & U_1 inaktiv (kurzgeschlossen)



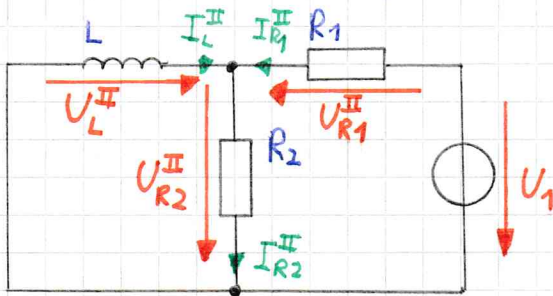
$$\underline{U}_0 = \underline{Z}_{\text{ges}}^I \cdot \underline{I}_{\text{ges}}^I = \underline{Z}_{\text{ges}}^I \cdot \underline{I}_L^I$$

$$\begin{aligned} \underline{Z}_{\text{ges}} &= \underline{Z}_L + \underline{Z}_{R1R2} = j\omega L + \frac{R_1 \cdot R_2}{R_1 + R_2} = \\ &= j 2\pi \cdot 10 \cdot 10^3 \text{ Hz} \cdot 159,155 \text{ H} \cdot 10^{-6} + \frac{12\Omega \cdot 30\Omega}{12\Omega + 30\Omega} = \\ &= \underline{13,17\Omega \angle 49,4^\circ} \end{aligned}$$

$$\underline{I}_L^I = \frac{\underline{U}_0}{\underline{Z}_{\text{ges}}^I} = \frac{10V}{13,17\Omega \angle 49,4^\circ} = \underline{0,76A \angle -49,4^\circ}$$

$$\begin{aligned} \underline{I}_{R2}^I &= \underline{I}_L^I \cdot \frac{R_1}{R_1 + R_2} = 0,76A \angle -49,4^\circ \cdot \frac{12\Omega}{12\Omega + 30\Omega} = \\ &= \underline{0,22A \angle -49,4^\circ} \end{aligned}$$

II) U_0 inaktiv & U_1 aktiv



$$\underline{U}_1 = \underline{Z}_{ges}^{\text{II}} \cdot \underline{I}_{ges}^{\text{II}} = \underline{Z}_{ges}^{\text{II}} \cdot \underline{I}_{R1}^{\text{II}} \quad \underline{Z}_L = j\omega L = j10\Omega \text{ (siehe I)}$$

$$\begin{aligned} \underline{Z}_{ges}^{\text{II}} &= \underline{Z}_{R1} + \underline{Z}_{R2L} = R_1 + \frac{j\omega L \cdot R_2}{j\omega L + R_2} = 12\Omega + \frac{j10\Omega \cdot 30\Omega}{j10\Omega + 30\Omega} \\ &= \underline{(15 + 9j)\Omega} = \underline{17,5 \angle 31^\circ} \end{aligned}$$

$$\underline{I}_{R1}^{\text{II}} = \frac{\underline{U}_1}{\underline{Z}_{ges}^{\text{II}}} = \frac{20V \angle 30^\circ}{17,5V \angle 31^\circ} = \underline{1,14A \angle -0,96^\circ}$$

$$\begin{aligned} \underline{I}_{R2}^{\text{II}} &= \underline{I}_{R1}^{\text{II}} \cdot \frac{\underline{Z}_L}{\underline{Z}_L + R_2} = 1,14A \angle -0,96^\circ \cdot \frac{10j\Omega}{10j\Omega + 30\Omega} \\ &= \underline{0,36A \angle 70,6^\circ} \end{aligned}$$

$$\begin{aligned} \text{I+II) } \underline{I}_{R2} &= \underline{I}_{R2}^{\text{I}} + \underline{I}_{R2}^{\text{II}} = 0,22A \angle -49,4^\circ + 0,36A \angle 70,6^\circ = \\ &= (0,261 + j0,176)A = \underline{0,315A \angle 34^\circ} \end{aligned}$$

b) $\underline{I}_0 = ?$

$$\underline{I}_0 = \underline{U}_0 \cdot \underline{I}_0^* \quad \underline{I}_0^{\text{I}} = \underline{I}_L^{\text{I}} \quad \underline{I}_0^{\text{II}} = \underline{I}_{R2}^{\text{II}} - \underline{I}_{R1}^{\text{II}}$$

$$\begin{aligned} \underline{I}_0 &= \underline{I}_0^{\text{I}} + \underline{I}_0^{\text{II}} = 0,76A \angle 49,4^\circ + 0,36A \angle 70,6^\circ - 1,14A \angle -0,96^\circ = \\ &= \underline{0,57A \angle -158^\circ} \end{aligned}$$

$$\underline{S}_0 = \underline{U}_0 \cdot \underline{I}_0^* = 10V \cdot 0,57A \angle +158^\circ = \underbrace{-5,29W}_P + j \underbrace{2,16\text{var}}_Q$$

alternativ:

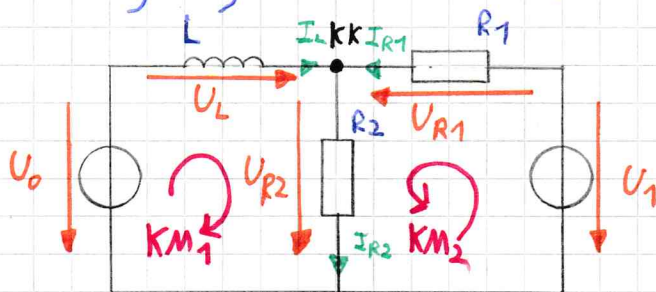
$$S_0 = U_0 \cdot I_0 = 10V \cdot 0,57A = \underline{5,71VA}$$

$$\varphi = \varphi_U - \varphi_I = 0^\circ - (-158^\circ) = \underline{158^\circ}$$

$$P = U_0 \cdot I_0 \cdot \cos \varphi = S \cdot \cos \varphi = 5,71 \cdot \cos(158^\circ) = \underline{-5,29W}$$

$$Q = U_0 \cdot I_0 \cdot \sin \varphi = S \cdot \sin \varphi = 5,71 \cdot \sin(158^\circ) = \underline{2,16\text{var}}$$

Lösungsweg 2: Maschen- & Knotengleichungen



$$\text{KK: } \underline{I}_{R2} = \underline{I}_L + \underline{I}_{R1}$$

$$\text{KM}_1: \underline{U}_0 = \underline{U}_L + \underline{U}_{R2}$$

$$\text{KM}_2: \underline{U}_1 = \underline{U}_{R1} + \underline{U}_{R2}$$

$$\underline{U}_0 = 10V$$

$$\underline{U}_1 = 20V \angle 30^\circ$$

$$\underline{U}_L = j \underbrace{2\pi \cdot 10\text{kHz} \cdot 159,155\text{H}}_{Z_L} \cdot \underline{I}_L$$

$$\underline{U}_{R1} = \underbrace{12\ \Omega}_{R1} \cdot \underline{I}_{R1}$$

$$\underline{U}_{R2} = 30\ \Omega \cdot \underline{I}_{R2}$$

⇓

8 Gleichungen mit 6 Unbekannten

⇓

Taschenrechner z.B. cSolve

⇓

$$\underline{I}_L = \underline{(-0,529 - 0,216j)A} = \underline{0,571A / -157,77^\circ}$$

$$\underline{I}_{R1} = \underline{(0,79 + 0,39j)A} = \underline{0,88A / 26,42^\circ}$$

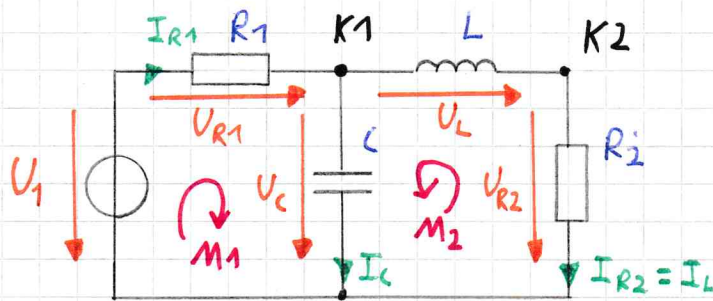
$$\underline{I}_{R2} = \underline{(0,261 + 0,176j)A} = \underline{0,315A / 34,01^\circ}$$

$$\underline{U}_L = \underline{2,16V - j5,29V} = \underline{5,71V / -67,77^\circ}$$

$$\underline{U}_{R1} = \underline{9,48V + j4,71V} = \underline{10,59V / 26,42^\circ}$$

$$\underline{U}_{R2} = \underline{7,84V + j5,29V} = \underline{9,45V / 34,01^\circ}$$

Bsp 2:



geg: $R_1 = 100 \Omega$ $R_2 = 20 \Omega$

$C_1 = 3,183 \text{ mF}$

$L_1 = 127,32 \text{ mH}$

$U_1 = 230 \text{ V}$ $f = 50 \text{ Hz}$

ges: a) verhält sich die Schaltung induktiv oder kapazitiv?

b) \underline{I}_1 bzw $\underline{I}_{\text{ges}}$

c) \underline{U}_{R1} & \underline{I}_{R2}

d) S, P & Q

e) λ

Lösungsweg 1: $\underline{Z}_{\text{ges}}$

a) $\underline{Z}_{R1} = R_1 = \underline{100 \Omega}$

$\underline{Z}_{R2} = R_2 = \underline{20 \Omega}$ mit $\omega = 2\pi f$

$\underline{Z}_{C1} = \frac{1}{j\omega C} = \frac{1}{j \cdot 2 \cdot \pi \cdot 50 \text{ Hz} \cdot 3,183 \text{ F} \cdot 10^{-6}} = \underline{-1000,03 j \Omega}$

$\underline{Z}_{L1} = j\omega \cdot L = j \cdot 2\pi \cdot 50 \text{ Hz} \cdot 127,32 \cdot 10^{-3} \text{ H} = \underline{40 j \Omega}$

$\underline{Z}_{R2L1} = \underline{Z}_{R2} + \underline{Z}_{L1} = \underline{20 \Omega + 40 j \Omega}$

$\underline{Z}_{C1R2L1} = \frac{\underline{Z}_{C1} \cdot \underline{Z}_{R2L1}}{\underline{Z}_{C1} + \underline{Z}_{R2L1}} = \underline{21,69 \Omega + 41,21 j \Omega}$

$\underline{Z}_{\text{ges}} = \underline{Z}_{R1} + \underline{Z}_{C1R2L1} = \underline{121,69 \Omega + 41,21 j \Omega}$

$$\operatorname{Im}\{Z_{\text{ges}}\} > 0 \Rightarrow \text{induktiv!}$$

$$b) \underline{I}_1 = \underline{I}_{\text{ges}} = \frac{\underline{U}_1}{Z_{\text{ges}}} = \underline{1,79 \text{ A } \angle -18,7^\circ}$$

$$c) \underline{U}_{R1} = Z_{R1} \cdot \underline{I}_1 = \underline{179 \text{ V } \angle -18,7^\circ}$$

$$\underline{I}_{R2} = \underline{I}_1 \cdot \frac{Z_{C1}}{Z_{C1} + Z_{R2L1}} = \underline{1,86 \text{ A } \angle -19,9^\circ}$$

$$d) \underline{S} = \underline{U}_1 \cdot \underline{I}_1^* = 230 \text{ V} \cdot 1,79 \text{ A } \angle +18,7^\circ =$$

$$= \underline{389,98 \text{ W} + j 132,07 \text{ var} = 411,73 \text{ VA } \angle +18,71^\circ}$$

P
Q
S
φ

$$e) \lambda = \frac{P}{S} = \cos(\varphi) = \underline{0,947}$$

Lösungsweg 2: Maschen- & Knoten Gleichungen

$$M1: \underline{U}_1 = \underline{U}_{R1} + \underline{U}_{C1}$$

$$M2: \underline{U}_{C1} = \underline{U}_L + \underline{U}_{R2}$$

$$K1: \underline{I}_{R1} = \underline{I}_C + \underline{I}_L \rightarrow \frac{\underline{U}_{R1}}{R_1} = \frac{\underline{U}_{C1}}{1/j\omega C} + \frac{\underline{U}_{L1}}{j\omega L}$$

$$K2: \underline{I}_L = \underline{I}_{R2} \rightarrow \frac{\underline{U}_L}{j\omega L} = \frac{\underline{U}_{R2}}{R_2}$$

„Bauteilgleichungen“ mit $\underline{U} = \underline{Z} \cdot \underline{I}$

4 Gl. mit 4 Unbekannten: \Rightarrow

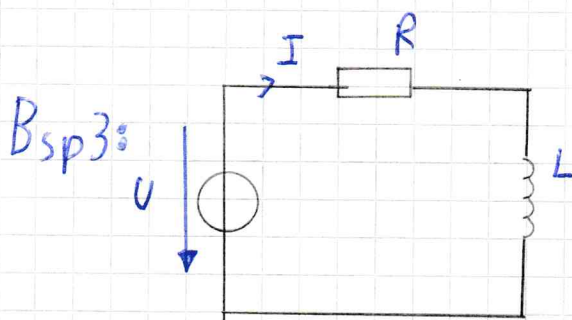
$$\underline{U}_{L1} = \underline{74,6 \text{ V } \angle 70^\circ} \quad \underline{U}_{C1} = \underline{83,4 \text{ V } \angle 43,53^\circ}$$

$$\underline{U}_{R1} = \underline{179,01V} \angle 18,7^\circ \quad \underline{U}_{R2} = \underline{37,25V} \angle -19,9^\circ$$

$$\underline{I}_{L1} = \frac{\underline{U}_{L1}}{\underline{Z}_{L1}} \quad \underline{I}_{R1} = \frac{\underline{U}_{R1}}{R_1}$$

$$\underline{I}_{C1} = \frac{\underline{U}_{C1}}{\underline{Z}_{C1}} \quad \underline{I}_{R2} = \frac{\underline{U}_{R2}}{R_2}$$

Rest wie bei Lösungsweg 1



geg: $P = 1,1 \text{ kW}$
 $U_1 = 230 \text{ V}$
 $\lambda_1 = \text{PF}_1 = 0,68$

ges: R, L, I_1

I_1 : $P = U \cdot I_1 \cdot \cos \varphi = U \cdot I_1 \cdot \lambda_1$

$$\Rightarrow I_1 = \frac{P}{U_1 \cdot \lambda_1} = \frac{1,1 \text{ kW}}{230 \text{ V} \cdot 0,68} = \underline{\underline{7,03 \text{ A}}}$$

$$\varphi_1 = \cos^{-1}(\lambda_1) = \underline{\underline{47,156^\circ}}$$

$$\varphi_{I_1} = \underline{\underline{-47,156^\circ}} \quad (\varphi = \varphi_U - \varphi_I \text{ mit } \varphi_U = 0^\circ)$$

$$\underline{\underline{I_1}} = \underline{\underline{7,03 \text{ A} \angle -47,156^\circ}} \quad \text{Netzstrom vor der Kompensation}$$

$$S_1 = U_1 \cdot I_1 = 230 \text{ V} \cdot 7,03 \text{ A} = \underline{\underline{1617,65 \text{ VA}}}$$

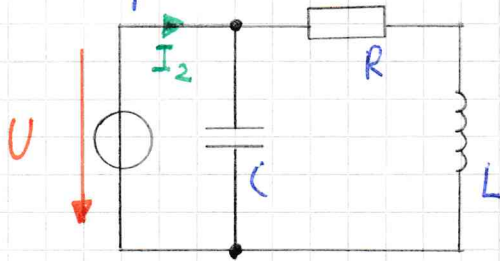
$$Q_1 = S_1 \cdot \sin(\varphi_1) = \underline{\underline{1186,08 \text{ var}}}$$

R, L: $U_1 = \underline{\underline{Z_1}} \cdot \underline{\underline{I_1}}$

$$\Rightarrow \underline{\underline{Z_1}} = \frac{U_1}{I_1} = \frac{230 \text{ V}}{7,03 \text{ A} \angle -47,156^\circ} = \underline{\underline{(22,2 + j24) \Omega}}$$

$$R_1 = \underline{\underline{22 \Omega}} \quad L_1 = \frac{Z_L}{j\omega} = \underline{\underline{76,3 \text{ mH}}}$$

mit Kompensation



$$\lambda_2 = 0,95$$

$$I_2 = \frac{P}{U_2 \cdot \lambda_2} = \frac{1,1 \text{ kW}}{230 \text{ V} \cdot 0,95} = \underline{\underline{5,034 \text{ A}}}$$

$$\varphi_{I_2} = -\varphi_2 = -\cos^{-1}(\lambda_2) = \underline{\underline{-18,19^\circ}}$$

$$\underline{\underline{I_2}} = \underline{\underline{5,034 \text{ A} \angle -18,19^\circ}}$$

$$S_2 = U_2 \cdot I_2 = 230 \text{ V} \cdot 5,034 \text{ A} = \underline{\underline{1157,82 \text{ VA}}}$$

$$Q_2 = S_2 \cdot \sin \varphi_2 = \underline{\underline{361,53 \text{ var}}}$$

$$Q_{\text{COMP}} = Q_{\text{DIFF}} = Q_2 - Q_1 = \underline{\underline{-824,55 \text{ var}}}$$

$$Q_c = \frac{U_c^2}{Z_c} \Rightarrow Z_c = \frac{U_c^2}{Q_c} = \frac{230^2 \text{ V}^2}{-j824,55 \text{ var}} = \underline{\underline{-j64,16 \Omega}}$$

$$Z_c = \frac{1}{j\omega C} \Rightarrow C = \frac{1}{j\omega Z_c} = 4,9 \cdot 10^{-5} \text{ F} = \underline{\underline{49 \mu\text{F}}}$$