

Übungen Mehrdimensionale Analysis am 20. 04. 2024

1.)

Bestimmen Sie die allgemeine Lösung der nachstehenden Differentialgleichung mit Hilfe der Variation der Parameter für die partikuläre Lösung.

$$y'' + 2y' + y = e^{-x} \cdot \ln(x)$$

2.) Lösen Sie die nachstehende inhomogene Differentialgleichung 4. Ordnung.

$$y'' - 4y' + 4y = 6 + \frac{e^{2x}}{x}$$

$$y(1) = 0$$

$$y'(1) = e^2 - 3$$

3.)

Lösen sie die nachstehende DGL 2. Ordnung.

$$y'' - 4y' + 5y = \frac{e^{2x}}{\sin(x)}$$

4.)

Lösen sie die nachstehende DGL 2. Ordnung.

$$y'' + 2y' + 2y = \frac{e^{-x}}{\cos^3(x)}$$

$$1.) \quad y'' + 2y' + y = e^{-x} \cdot \ln(x)$$

a) homogene

$$y'' + 2y' + y = 0$$

$$y = e^{\lambda x}$$

$$\lambda^2 + 2\lambda + 1 = 0$$

$$\lambda_{1,2} = -1 \pm \sqrt{1-1}$$

$$\underline{\underline{\lambda_1 = -1}}$$

$$\underline{\underline{\lambda_2 = -1}}$$

gleiche λ -Werte.

$$y_1 = e^{-x}$$

$$y_2 = x \cdot e^{-x}$$

$$\underline{\underline{y_h = A e^{-x} + B x e^{-x}}}$$

b) partikulär

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{-x} & x e^{-x} \\ -e^{-x} & e^{-x} - x e^{-x} \end{vmatrix}$$

$$W = e^{-2x} - x e^{-2x} + x e^{-2x} = \underline{\underline{e^{-2x}}}$$

$$y_p = u \cdot y_1 + v \cdot y_2$$

$$u = - \int \frac{r(x) \cdot y_2(x)}{w} dx = - \int \frac{e^{-x} \cdot \ln(x) \cdot x e^{-x}}{e^{-2x}} dx$$

$$u = - \int \frac{x \cdot \ln(x)}{1} dx = - \frac{x^2}{2} \ln(x) + \int \frac{x^2}{2} \cdot \frac{1}{x} dx$$

$$u = - \frac{x^2}{2} \ln(x) + \frac{1}{2} \int x dx = \underline{\underline{- \frac{x^2}{2} \ln(x) + \frac{x^2}{4}}}$$

$$v = \int \frac{r(x) \cdot y_1}{w} dx = \int \frac{e^{-x} \cdot \ln(x) \cdot e^{-x}}{e^{-2x}} dx$$

$$v = \int \frac{1 \cdot \ln(x)}{1} dx = x \ln(x) - \int x \cdot \frac{1}{x} dx$$

$$\underline{\underline{v = x \ln(x) - x}}$$

$$y_p = \left(- \frac{x^2}{2} \ln(x) - \frac{x^2}{4} \right) \cdot e^{-x} + (x \ln(x) - x) \cdot x e^{-x}$$

$$y_p = - \frac{x^2}{2} \ln(x) e^{-x} + \frac{x^2}{4} e^{-x} + x^2 \ln(x) e^{-x} - x^2 e^{-x}$$

$$\underline{\underline{y_{\text{all}} = A e^{-x} + B x e^{-x} + y_p}}$$

$$2.) \quad y'' - 4y' + 4y = 6 + \frac{e^{2x}}{x}$$

$$y(1) = 0 \quad y'(1) = e^2 - 3$$

1.) Homogen DGL.

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda_{1/2} = 2 \pm \sqrt{4-4}$$

$$\lambda_1 = 2$$

$$\rightarrow \underline{y_1 = e^{2x}}$$

$$\lambda_2 = 2$$

$$\rightarrow \underline{y_2 = x \cdot e^{2x}}$$

$$y_p = u \cdot y_1 + v \cdot y_2$$

$$W = y_1 y_2' - y_1' y_2 = e^{2x} \cdot (e^{2x} + 2x e^{2x}) - 2e^{2x} \cdot x e^{2x} = e^{4x} + \cancel{2x e^{4x}} - \cancel{2x e^{4x}} = \underline{\underline{e^{4x}}}$$

$$u = - \int \frac{y_2 \cdot r}{W} \cdot dx = - \int \frac{x e^{2x} \cdot \left(6 + \frac{e^{2x}}{x}\right)}{e^{4x}} dx$$

$$u = - \int \underbrace{6x}_{\frac{1}{8}} \cdot \underbrace{e^{-2x}}_{\frac{1}{2}} + 1 \, dx = - \left(6x \cdot \left(-\frac{1}{2}\right) e^{-2x} - \int 6 \cdot \left(-\frac{1}{2}\right) e^{-2x} dx + x \right)$$

$$= - \left(-3x e^{-2x} + \int 3 e^{-2x} dx + x \right) = - \left(-3x e^{-2x} - \frac{3}{2} e^{-2x} + x \right) = \underline{\underline{3x e^{-2x} + \frac{3}{2} e^{-2x} - x}}$$

$$v = \int \frac{y_1 \cdot r}{W} \cdot dx = \int \frac{e^{2x} \cdot \left(6 + \frac{e^{2x}}{x}\right)}{e^{4x}} dx = \int 6 e^{-2x} + \frac{1}{x} dx =$$

$$= \underline{\underline{-3 e^{-2x} + \ln(x)}}$$

$$y_p = u y_1 + v y_2 = \cancel{3x + \frac{3}{2}} - x e^{2x} - \cancel{3x + \ln(x)} \cdot x \cdot e^{2x} = \underline{\underline{\frac{3}{2} - x e^{2x} + x \ln(x) \cdot e^{2x}}}$$

$$\underline{\underline{y_{\text{allg}} = C_1 e^{2x} + C_2 x e^{2x} + \frac{3}{2} - x e^{2x} + x \ln(x) e^{2x}}}$$

$$y(1) = 0 = C_1 e^2 + C_2 e^2 + \frac{3}{2} - e^2$$

$$y'(x) = 2C_1 e^{2x} + C_2 e^{2x} + 2C_2 x e^{2x} - \cancel{e^{2x}} - \cancel{2x e^{2x}} + \cancel{\ln(x) e^{2x}} + \cancel{e^{2x}} + 2x \ln(x) e^{2x}$$

$$= 2C_1 e^{2x} + C_2 e^{2x} + 2C_2 x e^{2x} - 2x e^{2x} + \ln(x) e^{2x} + 2x \ln(x) e^{2x}$$

$$y'(1) = e^2 - 3 = 2C_1 e^2 + C_2 e^2 + 2C_2 e^2 - 2e^2$$

$$2C_1 e^2 + 3C_2 e^2 = 3e^2 - 3$$

$$C_1 e^2 + C_2 e^2 = e^2 - \frac{3}{2}$$

$$2C_1 + 3C_2 = 3 - 3e^{-2}$$

$$C_1 + C_2 = 1 - \frac{3}{2}e^{-2} \quad | \cdot 2 \quad \int -$$

$$C_2 = \cancel{3 - 3e^{-2}} - 2 + \cancel{3e^{-2}}$$

$$\underline{\underline{C_2 = 1}}$$

$$\underline{\underline{C_1 = -\frac{3}{2}e^{-2}}}$$

$$y(x) = -\frac{3}{2}e^{-2} \cdot e^{2x} + \cancel{x \cdot e^{2x}} + \frac{3}{2} - \cancel{x e^{2x}} + x \ln(x) e^{2x}$$

$$y(x) = -\frac{3}{2}e^{-2} e^{2x} + \frac{3}{2} + x \ln(x) e^{2x}$$

$$3) \quad y'' - 4y' + 5y = \frac{e^{2x}}{\sin(x)}$$

a) Wangere

$$\lambda^2 - 4\lambda + 5 = 0$$

$$\lambda_{1/2} = 2 \pm \sqrt{4-5}$$

$$\lambda_1 = 2+i$$

$$\lambda_2 = 2-i$$

$$\underline{y_h = e^{2x} \cdot (A \cos(x) + B \sin(x))}$$

$$y_1 = e^{2x} \cdot \cos(x)$$

$$y_2 = e^{2x} \cdot \sin(x)$$

$$W = \begin{vmatrix} e^{2x} \cos(x) & e^{2x} \sin(x) \\ 2e^{2x} \cos(x) - e^{2x} \sin(x) & 2e^{2x} \sin(x) + e^{2x} \cos(x) \end{vmatrix}$$

$$W = 2e^{4x} \sin(x) \cos(x) + e^{4x} \cos^2(x) - 2e^{2x} \sin(x) \cos(x) + e^{4x} \sin^2(x)$$

$$\underline{W = e^{4x}}$$

$$u = - \int \frac{f(x) \cdot y_2(x)}{W} dx = - \int \frac{e^{2x} \cdot e^{2x} \sin(x)}{\sin(x) \cdot e^{4x}} dx = \underline{\underline{-x}}$$

$$V = \int \frac{g(x) \cdot y_1(x)}{\omega} dx = \int \frac{e^{2x} \cdot e^{2x} \cdot \omega(x)}{\sin(x) \cdot e^{4x}} dx = \underline{\underline{\ln(\sin(x))}}$$

$$\underline{y_p = -x \cdot e^{2x} \cdot \omega(x) + \ln(\sin(x)) e^{2x} \sin(x)}$$

$$y_{\text{all}} = y_h + y_p$$

$$4.) \quad y'' + 2y' + 2y = \frac{e^{-x}}{\cos^3(x)}$$

a) homogene.

$$r^2 + 2r + 2 = 0$$

$$r_{1,2} = -1 \pm \sqrt{1-2}$$

$$\underline{r_1 = -1 + i}$$

$$\underline{r_2 = -1 - i}$$

$$y_1 = e^{-x} \cos(x)$$

$$y_2 = e^{-x} \sin(x)$$

$$\underline{y_h = C_1 e^{-x} \cos(x) + C_2 e^{-x} \sin(x)}$$

b) partikul.

$$W = \begin{vmatrix} e^{-x} \cos(x) & e^{-x} \sin(x) \\ -e^{-x} \cos(x) - e^{-x} \sin(x) & -e^{-x} \sin(x) + e^{-x} \cos(x) \end{vmatrix}$$

$$= -e^{-2x} \cancel{\sin(x) \cos(x)} + e^{-2x} \cos^2(x) + e^{-2x} \cancel{\sin(x) \cos(x)} + e^{-2x} \sin^2(x)$$

$$\underline{W = e^{-2x}}$$

$$y_p = u \cdot y_1 + v \cdot y_2$$

$$u = - \int \frac{g(x) \cdot y_2(x)}{\omega} dx$$

$$u = - \int \frac{e^{-x} \cdot e^{-x} \sin(x)}{\omega^3(x) \cdot e^{-2x}} dx = - \int \frac{\sin(x)}{\omega^3(x)} dx$$

$$u = - \int f(x) \cdot \frac{1}{\omega^2(x)} dx = \underline{\underline{-\frac{1}{2} \ln^2(x)}}$$

$$v = \int \frac{g(x) \cdot y_1}{\omega} dx = \int \frac{e^{-x} \cdot e^{-x} \omega(x)}{\omega^3(x) \cdot e^{-2x}} dx = \underline{\underline{f(x)}}$$

$$\underline{\underline{y_p = -\frac{1}{2} \ln^2(x) e^{-x} \omega(x) + f(x) e^{-x} \sin(x) = \frac{1}{2} f(x) \cdot \sin(x) e^{-x}}}$$

$$y_{\text{allg}} = y_h + y_p$$